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The Variable Elasticity of Substitution Function and Endogenous Growth: An Empirical Evidence from Vietnam

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Abstract:

Purpose: *To specify a Variable Elasticity of Substitution function (VES), in which the estimated Elasticity of Substitution (ES) can give some implications for the tendency of economic growth in the Vietnames manufacturing sector.*
Design/Methodology/Approach: *The contribution and the relevant methodology is based on the Bayesian approach having some advantages over the frequentist method: (i) the simulation and prediction results are more reliable in Bayesian analysis due to combining prior knowledge about parameters with obverved data to compose a posterior model, whereas the frequentist approach is based only on available data; (ii) in probability sense, Bayesian credible intervals have a straightforward interpretation compared to frequentist confidence intervals. The Bayesian nonlinear regresion performed is suitable for fitting production functions and depicting economic growth.*
Findings: *The specified VES function has the ES greater than one and this finding contradicts many previous empirical studies in the growth theory. This result points to the possibility of unbounded endogenous growth in the Vietnamese manufacturing sector.*
Practical implications: *Based on the empirical results, in order to realize the possibility of endogenous growth for the studied Vietnamese manufacturing sector, policies of enforcing investment are needed. To raise the level of science and technique, as well as human capital of the Vietnamese enterprises, at the same time, there is great necessity to encourage R&D activities in both the private and public sectors.*
Originality/Value: *Although this study organically builds upon recent studies about the link between the VES, the elasticity of factor subsitution and economic growth, its results proved that the VES is more appropriate than the Cobb-Douglas and the Constant Elasticity of Substitution (CES) to explain economic growth in the view of capital-labor relationship.*

Keywords: *VES function, elasticicty of factor substitution, endogenous growth, Bayesian nonlinear regression, CES.*

JEL Codes: *C11, O41, O47.*

Paper Type: *Research Paper.*

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1. Introduction

As it is well-known in growth literature a significant role belongs to the Elasticity of Substitution (ES). The effects of the ES are multidimensional and complex. The ES influences the pace of convergence towards the balanced growth path (Klump and Preissler, 2000). It affects the income distribution in an economy (Hicks, 1932). It can trigger changes in the savings ratio during the transition (Smetter, 2003). More specifically, it is one of the determinants of economic growth (de La Grandville, 1989; Klump and de La Grandville, 2000), particularly, it potentially creates the possibility of unbounded endogenous growth (Solow, 1956; Jones and Manuelli, 1990; 1997; Barro and Sala-i-Martin, 1995; Pavilos and Karagiannis, 2004; Karagiannis *et al.*, 2005; Thalassinos and Stamatopoulos, 2015).

The overwhelming majority of previous studies on economic growth constructed growth models, where technology is described by the Cobb-Douglas function, which has extremely rigid presumptions, namely its ES is equal to unity. Therefore, the Constant Elasticity of Substitution (CES) with the ES constant but different from one showed up in 1961 by Arrow *et al.* (1961). This research direction in growth theory was followed by, among others, Brown and DeCani (1963), Ferguson (1965), Sato (1970), Revankar (1971b), McFadden (1978), Thach (2020).

Nevertheless, the CES does not account for the interaction between the ES and the level of economic development. For this reason, the VES was established in 1971. Revankar (1971a) first introduced this functional form. The VES and the CES have some same properties, except for the case where the ES is constant along an isoquant for the CES, but for the VES it is constant only along a ray from the origin. Compared to the CES, the VES allows for analyzing the impact of a change in an economy's per capita capital on the ES between capital and labor. This change, in turn, feeds back in the economy affecting investment and output growth. From this, the VES model exhibits the possibility of unbounded endogenous growth in spite of the absence of exogenous technical progress and the presence of non-reproducible inputs (Thalassinos *et al.*, 2012).

In Vietnam, most of the work explored the Cobb-Douglas and its variants (Tu and Nguyen, 2012; Nguyen, 2013; Khuc and Tran, 2016; Pham and Ly, 2016; Huynh, 2019) within the frequentist framework. It is noted that the Cobb-Douglas has the unitary ES, that hides the crucial role of the ES in the economic growth process. Hence, the current study uses a panel of 227 Vietnamese manufacturing companies over an 11-year period to specify a VES function. The estimated ES in the sample is larger than one, which provides conclusive evidence on unbounded endogenous growth in this sector. As this sector plays a leading role in the production activities of the country, this finding will offer strong implications for the overall economic growth.

The remainder of the article is structured as follows. Section 2 introduces the Revankar (1971a; 1971b) VES specification. Empirical studies on the VES are overviewed in Section 3. Section 4 discusses the data, estimation methods, model specification. Empirical results and discussions are provided in Section 5. Section 6 includes.

2. A VES Production Function

2.1 The Revankar Specification

Resting on Revankar (1971a) and recently Sato and Hoffman (1961), Karagiannis *et al.* (2005), we consider the VES specification as follows:

$$Y = AK^{\alpha\varepsilon}[L + \beta\alpha K]^{(1-\alpha)\varepsilon}, \qquad (1)$$

where Y, K, and L represent output, capital, and labor, respectively, A is efficiency parameter, ε devotes returns to scale, α, β are parameters.

Assuming that $\varepsilon = 1$, let us write this production function in intensive form:

$$y = Ak^{\alpha}[1 + \beta\alpha k]^{(1-\alpha)}, \qquad (2)$$

where $y = f(k), y \equiv \frac{Y}{L}, k \equiv \frac{K}{L}$.

Differentiating (1), we obtain:

$$f'^{(k)} = \alpha\frac{y}{k} + \alpha(1-\alpha)\beta\frac{y}{1+\alpha\beta k}. \qquad (3)$$

The second-order differentiation of (2) gives the following:

$$f''^{(k)} = A\alpha(1-\alpha)(1+\alpha\beta k)^{-\alpha-1}k^{-1}.\ (4) \qquad (4)$$

Note that this function satisfies the properties of a neoclassical production function, namely:

$f(k) > 0, f'(k) > 0$, and $f''^{(k)} < 0\ \forall k > 0$, as long as $A > 0, 0 < \alpha \leq 1, \beta > -1$ and $k^{-1} > -b$.

In case of $\beta = 0$, equation (2) reduces to the Cobb-Douglas function, while if $\alpha = 1$, then it reduces to the Ak case.

2.2 Main Properties of the VES Function

Equation (2) has the following limiting properties:

$$\lim_{k\to 0} f(k) = 0, \lim_{k\to\infty} f(k) = \infty \text{ if } \beta > 0,$$

$$\lim_{k\to -\beta^{-1}} f(k) = A(-\beta)^{-\alpha}(1-\alpha)^{1-\alpha} > 0 \text{ if } \beta < 0. \tag{5}$$

Next, equation (3) leads to the following:

$$\lim_{k\to 0} f'(k) = \infty, \lim_{k\to\infty} f'(k) = A(\alpha\beta)^{1-\alpha} > 0 \text{ if } \beta > 0,$$

$$\lim_{k\to -\beta^{-1}} f'(k) = A[-\beta(1-\alpha)^{1-\alpha} > 0 \text{ if } \beta < 0. \tag{6}$$

Hence, in case $\beta > 0$, one of the Inada conditions is violated, concretely the marginal returns to capital are strictly bounded from below and it means that labor is not essential input. In other words, if $\beta > 0$, then

$$\lim_{L\to 0} F(K,L) = (\alpha\beta)^{1-\alpha} > 0.$$

From equation (2), the labor share is expressed as:

$$s_L = \frac{1-\alpha}{1+\alpha\beta k}, \quad where \lim_{k\to 0} s_L = 1-\alpha,$$

$$\lim_{k\to\infty} s_L = 0 \text{ if } \beta > 0 \text{ and } \lim_{k\to -\beta^{-1}} s_L = 1 \text{ if } \beta < 0. \tag{7}$$

And the capital share is:

$$s_K = 1 - s_L = \frac{\alpha + \alpha\beta k}{1+\alpha\beta k}.$$

Thus, the ES in equation (2) is calculated as follows:

$$\sigma(k) = 1 + \beta k > 0. \tag{8}$$

In general, the ES denotes how the ratio of inputs changes if the marginal rate of technical substitution between them varies by one percent (Thach, 2020). In other words, the ES is a measure of the ease of substitution between capital and labor.

In sum, $\sigma > 1$ if $\beta > 0$ and $\sigma < 1$ if $\beta < 0$. In other words, the ES varies with per capita capital level as an indicator of economic development. It is noted that the ES, in turn, affects the development process. According to the findings of Jones and Manuelli (1990; 1997), unbounded endogenous growth can take place in spite of the absence of exogenous technical progress and the presence of non-reproducible inputs as long as the marginal returns to capital are strictly bounded from below, while results in Palivos and Karagiannis (2004) ensured that the ES becoming asymptotically (as k increases) higher than one is necessary and sufficient for the existence of a lower bound on the marginal returns to capital. So, as long as $\sigma > 1$, with growing k, the model exhibits the possibility of unbounded endogenous growth.

3. Empirical Research on the VES

Since the VES function appeared, it has been applied more and more widely. Previous studies using this function can be classified into two groups: time-series models and models using cross-section data.

Among others, Sato and Hoffman (1968), Lovell (1968), Revankar (1971b), Lovell (1973a), Roskamp (1977) and Bairam (1989; 1990) enter the former group. Sato and Hoffman (1968) based on analysis of a dataset from the Japanese and American private non-farm sector stated that in general, the VES fits better than the CES. Also testing with the private non-farm data of Japan, Revankar (1971b) rejected the Cobb-Douglas in favor of the VES. Investigating the U.S. manufacturing sector, Lovell (1973a) accepted both the CES and the VES. However, for 16 two-digit U.S. manufacturing industries Lovell (1968) rejected the Cobb-Douglas and the CES in favor of the VES. Bairam (1989; 1990) researching the Japanese and Soviet economies rejected the Cobb-Douglas in favor of the VES. Roskamp (1977) using a sample from 38 Germany manufacturing industries specified the ES for both the CES and the VES. Note that most of these studies provided the estimates of the ES lower than one, except for Roskamp (1977) in 7 out of 38 industries and for Bairam (1989).

The latter group includes, among others, Lu and Fletch (1968), Revankar (1971a), Lovell (1973b), Kazi (1980), Diwan (1970), Meyer and Kadiyala (1974), Tsang and Yeung (1976), Zellner and Ryu (1998). Studying formally the U.S. two-digit manufacturing industries, Lu and Fletcher rejected the CES in favor of the VES in 7 to 9 out of 17 industries. Also using the data from these industries, Revankar (1971a) rejected the Cobb-Douglas in favor of the VES in 5 out of 12 industries. Lovell (1973b) rejected both the Cobb-Douglas and the CES in favor of the VES in 3 out of 17 U.S. two-digit manufacturing sectors. Kazi (1980) rejected the CES in favor of the VES for most cases. Making use of micro data for U.S. two-digit manufacturing firms, Diwan (1970) also rejected the Cobb-Douglas and the CES in favor of the VES. Utilizing agricultural experimental data, Meyer and Kadiyala (1974) gave the same results. Without model comparison, Tsang and Yeung (1976), Zellner and Ryu

(1998) estimated the CES and VES specifications for the food and kindred products and transportation equipment industries in the U.S. It is noted that these cross-section investigations provided estimates of the ES less than one, except for Lu and Fletcher (1968) and for Kazi (1980).

As mentioned above, to the author's best knowledge, most studies in Vietnam used the Cobb-Douglas and its modifications, which always have the ES equal to one. In Tu and Nguyen (2012), the Cobb-Douglas is used to consider the effect of production factors on coffee productivity in Dak Lak province. Applying the accounting method, Nguyen (2013) specified a Cobb-Douglas function to identify the resources of the economic growth of Hung Yen province. Khuc and Tran (2016) constructed an extended Cobb-Douglas function to identify inputs contributing to the industry growth of Vietnam. Using the accounting method, Le (n.d) estimated a Cobb-Douglas function for Vietnam from data of mining, processing industry, electricity and water production and distribution enterprises. According to the results obtained, the proportion of labor and fixed capital in the gross output of the studied industries varies a range from 0.11 to 0.39 and 0.89 to 0.61, respectively. With regard to variants of the Cobb-Douglas, in Pham and Ly (2016), a translog Cobb-Douglas function is specified for the Vietnamese manufacturing enterprises based on the data extracted from the 2010 Vietnam Enterprise Survey by the General Statistics Office. Huynh (2019) applying the MLE method to a dataset taken from the Enterprise Survey of the General Statistics Office for the 2013-2016 period to estimate a Battese-Coelli production function.

4. Methodology and Data

4.1 Research Methods and Model Specification

Note that in most previous investigations, the Cobb-Douglas, CES or VES functions were specified within the frequentist approach. Still, since the 1990s, the Bayesian framework has applied commonly in social research due to its big advantages over the frequentist statistics (Anh, 2018; Briggs and Hung, 2019; Thach *et al.*, 2019; Hung *et al.*, 2019; Kreinovich, 2019; Thach, 2019; Thach, 2020). First, the estimation results are more reliable in Bayesian analysis because of the combination of prior information about parameters with observed data to fit a posterior model, whereas frequentist analysis is based only on available data. Second, in probability sense, Bayesian credible intervals have a straightforward interpretation compared to frequentist confidence intervals. In Bayesian analysis, conditional probability $p(A|B) = \frac{p(A,B)}{p(B)}$ is used to derive Bayes's theorem:

$$p(B|A) = \frac{p(A|B)p(B)}{p(A)}, \tag{9}$$

where A, B are random vectors.

Let us assume that a data vector X is a sample from a probability model with the unknown parameter vector θ. Using a likelihood function, we formulate this model as follows:

$$L(\theta; X) = f(X; \theta) = \prod_{i=1}^{n} f(X_i|\theta), \quad (10)$$

where $f(X_i|\theta)$ is a probability density function of X given θ.

Given data X, we infer the properties of θ. In Bayesian models, parameter θ is a random vector.

The Bayesian analysis begins with specifying a posterior model. The posterior distribution consists of two components: a likelihood function incorporating information about the model parameters based on available data and prior distribution containing known information about the model parameters. The likelihood function and priors are combined by the Bayes law to specify a posterior model:

$$\text{Posterior} \propto \text{Likelihood} \times \text{Prior}. \quad (11)$$

Since both X and θ are random variables, Bayes's theorem is used to acquire the posterior distribution of θ given X:

$$p(\theta|X) = \frac{p(X|\theta)p(\theta)}{p(X)} = \frac{f(X;\theta)\pi(\theta)}{m(X)}, \quad (12)$$

where $m(X) \equiv p(X)$ is known as the marginal distribution of X which is expressed as follows:

$$m(X) = \int f(X; \theta)\pi(\theta)d(\theta), \quad (13)$$

where $f(X; \theta)$ is a likelihood function of X given θ, π(θ) denotes a prior distribution for θ, $m(X)$ presents prior predictive distribution.

In the present work, we consider both cases of non-constant ($\varepsilon \neq 1$) and constant ($\varepsilon = 1$) returns to scale. For this, taking natural logs of both sides of (1), we yield the estimation equations:

$$lnY_{ij} = b0 + \alpha\varepsilon lnlnK_{ij} + (1-\alpha)\varepsilon \ln(L_{ij} + \beta\alpha K_{ij}) + \epsilon_{ij}, \tag{14}$$

and

$$lnY_{ij} = b0 + \alpha lnK_{ij} + (1-\alpha)\ln(L_{ij} + \beta\alpha K_{ij}) + \epsilon_{ij} \tag{15}$$

where $lnY_{ij}, lnK_{ij}, K_{ij}$ and L_{ij} present natural log of output, natural log of capital, capital, and labor employed, respectively, $b0$ stands for lnA, i denotes years, j denotes companies, ϵ_{ij} is a random error. The conditions $0 < \alpha \leq 1, \beta > -1$ must be satisfied.

In this research, the author aims to specify two VES functions: restricted ($\varepsilon = 1$) and unrestricted ($\varepsilon \neq 1$). To achieve this purpose, we perform the Bayesian nonlinear regression. According to the experience of specifying Bayesian models, in the absence of previous studies or in the presence of a large sample size available, we can choose weakly informative or noninformative prior distributions for model parameters. In our case, the neoclassical propositions and sufficient data suggest which priors to select for our models. Therefore, we assign the normal N(1,100) prior to parameter $b0$, the normal N(0,100) prior to parameter ε, the uniform(0,1) prior to parameter α, the gamma(1,1) prior to parameter β, and the Igamma(0.001, 0.001) prior to the overall variance ($sig2\_0$).

Based on equations (14) and (15), our two models are specified in a similar way. The difference between them is that parameter ε is discarded in model 2. Hence:

Model 1:

$$lny2010_{ij} = b0 + \alpha\varepsilon lnk2010_{ij} + (1-\alpha)\varepsilon\ln(l_{ij} + \beta\alpha k2010_{ij})$$

$$b0 \sim N(1, 100)$$

$$\varepsilon \sim N(0, 100)$$

$$\alpha \sim uniform(0, 1)$$

$$\beta \sim gamma(1, 1)$$

$$sig2\_0 \sim Igamma(0.001, 0.001) \tag{16}$$

Model 2:

$$lny2010_{ij} = b0 + \alpha lnk2010_{ij} + (1-\alpha)\ln\left(l_{ij} + \beta\alpha k2010_{ij}\right) + \epsilon_{ij}$$

$$b0 \sim N(1, 100)$$

$$\alpha \sim uniform(0, 1)$$

$$\beta \sim gamma(1, 1)$$

$$sig2\_0 \sim Igamma(0.001, 0.001) \tag{17}$$

where $lny2010_{ij}$, $lnk2010_{ij}$, $k2010_{ij}$, and l_{ij} present natural log of output, natural log of capital, capital, and labor employed, respectively, year i=2008,…, 2018, company j=1, 2, 3,…, 227.

4.2 Data Description

The research utilizes a panel obtained from the financial statements and annual reports of 227 non-financial companies listed at Ho Chi Minh Stock Exchange and Ha Noi Stock Exchange in Vietnam over the period 2008 to 2018. The dataset consists of 1,974 observations. Net revenue and net fixed assets stand for output and capital. Calculation of net revenue and net fixed assets relies on the 2010 producer price index (PPI) (Vietnam General Statistics Office, 2018). Revenue, assets, and labor are measured in million VND, million VND, and the number of employees, respectively. Utilizing a database of the listed companies enables labor and capital shares not to be skewed due to statistical errors that often appear when the mixed incomes from households' labor and capital contributions as well as those in the state-owned sector are used. Look at the measurements of the model variables demonstrated in Table 1. Descriptive statistics in Table 2:

Table 1. *The measurements of the variables*

Variable	Notation	Measurement	Source
Labor	Lnl	Natural log (number of persons)	Companies' annual report
Capital	Lnk2010	Natural log (net fixed assets/PPI)	Companies' financial statement
Output	Lny2010	Natural log (net revenue/ PPI)	Companies' financial statement

Table 2. *Descriptive Statistics*

Variable	Obs	Mean	Std. Dev.	Min	Max
lny2010	1,974	13.21938	1.364355	8.579272	17.50561
lnk2010	1,974	11.5871	1.64275	5.599666	16.93655
l	1,974	1185.77	1793.31	17	19828
k2010	1,974	497569.7	1614555	270.336	2.27e+07
ppi2010	1,974	124.1931	13.89073	99.08	139.43

5. Results of Bayesian Simulations

Firstly, we compare the two candidate models: unrestricted model 1 ($\varepsilon \neq 1$) and restricted model 2 ($\varepsilon = 1$). These are Bayesian nonlinear models and Bayesian information criteria and Bayesian model tests are applied to their comparison (Tables 3 and 4). The results of the model comparison show that compared to model 2, model 1 has less DIC value, greater log(ML) and log(BF) level (Table 3), and higher posterior probability (Table 4). According to these results, model 1 is more appropriate and we choose this model to proceed to the Bayesian inference stage.

Table 3. *Bayesian information criteria*

	DIC	log(ML)	log(BF)
m1	4500.096	-2299.369	.
m2	4683.485	-2362.177	-62.80816

Table 4. *Bayesian model tests*

	log(ML)	P(M)	P(M|y)
m1	-2.30e+03	0.5000	1.0000
m2	-2.36e+03	0.5000	0.0000

Secondly, in Bayesian modeling, tests for model robustness should be performed before inference. Hence, we conduct convergence diagnostics for MCMC chains of model 1. At first, look at the acceptance rate and efficiency of MCMC sampling in Bayesian models. Acceptance rate is defined as the number of proposals accepted in the total proposals, whereas efficiency means the mixing properties of MCMC sampling. Both of these rates affect MCMC convergence. Furthermore, we turn to convergence check. There are two main methods of convergence inspection: graphical and formal. We shall examine trace plots, autocorrelation plots, cusum plots and histogram plots to monitor the behavior of MCMC chains.

Roberts and Rosenthal (2001) supposed that optimal acceptance rates range approximately from 0.15 to 0.5. For model 1 selected, the smallest, average and largest efficiencies obtain 0.011; 0.238 and 1, which are higher than a warning level of 0.01, whereas those of model 2 are about 0.037; 0.279 and 1. So, the MCMC sampling of model 1 and model 2 has reached the same acceptance rate (0.54), which is acceptable in view of Roberts and Rosenthal (2001). The smaller MC errors (MCSE) of the posterior mean are, the more accurate the estimates. In our case, all these values of model 1 and model 2 are close to one decimal, which is reasonable for MCMC algorithms (Table 5).

In Bayesian analysis, credible intervals have a straightforward probability interpretation. For instance, for our model 1, the probability of the posterior mean of parameter alpha in the interval (0.07; 0.44) is 95% (Table 5a). For model 2 Table 5b.

Table 5. *Estimation results for the specified models*

a. Model 1

	Mean	Std. Dev.	MCSE	Median	Equal-tailed [95% Cred.	Interval]
alpha	.2555044	.0957007	.012609	.2590413	.0724041	.4405073
b0	6.409894	.2872717	.03529	6.415377	5.822879	6.927493
beta	.0264373	.0279579	.003463	.0173696	.0039695	.1110825
epsilon	.8224508	.0148042	.000534	.822479	.7940699	.8521968
sig2_0	.5879223	.0187877	.000266	.5874715	.5535147	.6265438

b. Model 2

	Mean	Std. Dev.	MCSE	Median	Equal-tailed [95% Cred.	Interval]
alpha	.3906046	.0499093	.00365	.3950448	.2834035	.4746916
b0	4.67459	.1983853	.014362	4.664354	4.326251	5.090772
beta	.0021382	.001619	.000113	.0017465	.0003492	.0061079
sig2_0	.6270789	.0199078	.000282	.6264413	.588852	.6664539

As mentioned above, MCMC convergence should be tested before Bayes inference stage, for estimation results are robust only when MCMC chains converge to a stationary distribution. The results recorded in Figure 1 indicate that regarding model 1, the diagnostic graphs are reasonable. The trace plots show no trends, traversing rather quickly through the distribution; the autocorrelation plots die off after 1-86 lags, which can be considered acceptable; the histograms resemble the shape of probability distributions (Figure 1).

CUSUM plots provide an additional visual method for checking MCMC convergence (Figure 1). For model 1 and model 2, the CUSUM lines are jagged, which certainly indicates MCMC convergence (Figure 1). Generally, the MCMC chains of both models mix well. We can conclude that our simulations face no serious convergence problem and the MCMC chains have converged to the target distribution. It is noted that although model 1 is more appropriate than model 2, for the latter, MCMC chains seemingly mix somewhat better.

Besides graphical diagnostics, common formal tests, such as an effective sample size can be applied (Table 6). If the level of efficiency is more than one, then it is a satisfactory result. In Table 6, the efficiency of all the parameters of both models is more than 0.01, which also shows no sign of non-convergence.

Figure 1. *Graphical diagnostics for MCMC convergence (model 1)*

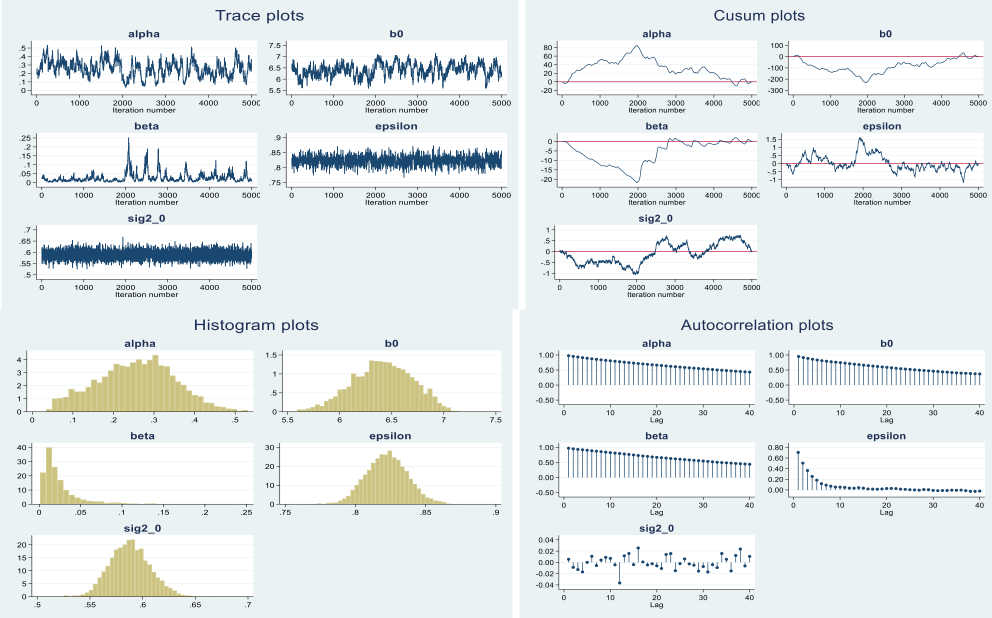

Table 6. *Effective sample size*

a. Model 1

	ESS	Corr. time	Efficiency
alpha	57.60	86.80	0.0115
b0	66.26	75.46	0.0133
beta	65.18	76.71	0.0130
epsilon	767.44	6.52	0.1535
sig2_0	5000.00	1.00	1.0000

b. Model 2

	ESS	Corr. time	Efficiency
alpha	186.94	26.75	0.0374
b0	190.81	26.20	0.0382
beta	205.93	24.28	0.0412
sig2_0	5000.00	1.00	1.0000

5.1 Results of the VES Specifications

According to the above results of model comparison, model 1, an unrestricted model, was chosen for further inference. So, let us perform further analysis of this model. We are the most interested in variables beta (β) and epsilon (ε). First, compared to model 2, where returns to scale equal one, in model 1, returns to scale are equivalent to 0.8. Second, because β =0.26>0, from the formula of the ES in (8) we can see that $\sigma(k) = 1 + \beta k > 0$. This finding leads to an important conclusion that Vietnamese manufacturing companies have the possibility of unbounded endogenous growth. Possible explanations for this are that in the period after the Great recession 2008-2009, the Vietnamese economy has achieved rather fast restoration thanks to the manufacturing companies' expanding investment. For key manufacturing industries, investment growth rate reached 6.5% in 2012, 9.15 in 2013, 14% in 2014, 7.2% in 2015, 7% in 2016, 9.5% in 2017, and 8.9% (preliminary estimate) in 2018 (Vietnam General Statistics Office, 2019). Huge investment increases the ES, which, in turn, has made the possibility of unbounded endogenous growth probable.

6. Conclusion

The present study was conducted to specify a Revankar (1971) VES function for the Vietnamese manufacturing sector including 227 companies in the sample. The great advantage of the VES has over the Cobb-Douglas as well as the CES, that is allowing for analysis of the relationship between the ES and the level of economic development, in particular, the level of the ES indicates the possibility of unbounded endogenous growth. By using the Bayesian non-linear regression, empirical results demonstrate that the estimated ES is greater than one for both cases of constant and non-constant returns to scale.

Moreover, the results of the model comparison indicate that the unrestricted model is more appropriate. Specifically, contrary to Thach (2020), where the estimated result provided an ES lower than one in a specified CES function, in this work, the estimated ES is higher than one in the VES specification, which points to the possibility of unbounded endogenous growth in the Vietnamese manufacturing sector. In other words, as long as the ES is higher than unity, with increasing capital-labor ratio, this business sector can perform unbounded endogenous growth. As this sector is a leading driver of economic growth, our research results provide good implications for the overall economic growth of Vietnam. Hence, in order to realize the possibility of endogenous growth for the studied Vietnamese companies, it is necessary to carry out policies of enforcing investment. Moreover, the level of science and technique, as well as human capital, is low in Vietnamese enterprises, so at the same time, there is great necessity to enhance R&D activities in both the private and public sectors.

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